

Foundation GCSE: Column Vectors

Name: _____

Class: _____

Instructions: Answer all questions. Questions are ordered from easiest (1) to hardest (20). Working out should be shown where appropriate.

1. Given vector $\mathbf{a} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$, calculate $\mathbf{a} + \mathbf{b}$.
2. Given $\mathbf{p} = \begin{pmatrix} 5 \\ -2 \end{pmatrix}$ and $\mathbf{q} = \begin{pmatrix} -1 \\ 4 \end{pmatrix}$, calculate $\mathbf{p} + \mathbf{q}$.
3. Given $\mathbf{u} = \begin{pmatrix} 7 \\ 8 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$, calculate $\mathbf{u} - \mathbf{v}$.
4. Given $\mathbf{a} = \begin{pmatrix} -3 \\ 6 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$, calculate $\mathbf{a} - \mathbf{b}$.
5. Find $3\mathbf{a}$ given $\mathbf{a} = \begin{pmatrix} 2 \\ -5 \end{pmatrix}$.
6. Calculate $-2\mathbf{v}$ given $\mathbf{v} = \begin{pmatrix} -4 \\ 3 \end{pmatrix}$.
7. Given $\mathbf{x} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$, calculate $2\mathbf{x} + \begin{pmatrix} 1 \\ -2 \end{pmatrix}$.
8. Given $\mathbf{p} = \begin{pmatrix} -2 \\ 5 \end{pmatrix}$ and $\mathbf{q} = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$, calculate $3\mathbf{p} - 2\mathbf{q}$.
9. Calculate the magnitude of the vector $\mathbf{a} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$.
10. Calculate the magnitude of the vector $\mathbf{b} = \begin{pmatrix} 5 \\ -12 \end{pmatrix}$.
11. Find vector $\mathbf{x}$ if $\mathbf{x} + \begin{pmatrix} 2 \\ -3 \end{pmatrix} = \begin{pmatrix} 5 \\ 4 \end{pmatrix}$.
12. Find vector $\mathbf{y}$ if $2\mathbf{y} - \begin{pmatrix} 4 \\ 6 \end{pmatrix} = \begin{pmatrix} 6 \\ 2 \end{pmatrix}$.
13. Given points $A(1, 2)$ and $B(4, 6)$, write down the column vector $\overrightarrow{AB}$.
14. Given points $P(-2, 3)$ and $Q(3, -9)$, write down the column vector $\overrightarrow{PQ}$.
15. Calculate the magnitude of the vector $\mathbf{c} = \begin{pmatrix} -6 \\ 8 \end{pmatrix}$.
16. Given $\mathbf{a} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$, and $\mathbf{c} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$, evaluate $2\mathbf{a} - \mathbf{b} + 3\mathbf{c}$.
17. Find the magnitude of vector $\overrightarrow{AB}$ where $A(2, -1)$ and $B(5, 3)$. Give your answer as an exact value.
18. Vector $\mathbf{m} = \begin{pmatrix} k \\ 6 \end{pmatrix}$ has a magnitude of 10. Find the positive value of k .
19. Given $\mathbf{u} = \begin{pmatrix} x \\ 3 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 4 \\ y \end{pmatrix}$, and that $\mathbf{u} + \mathbf{v} = \begin{pmatrix} 7 \\ -2 \end{pmatrix}$, find the values of x and y .
20. Vector $\mathbf{a} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$ and vector $\mathbf{b} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$. Find the magnitude of the vector $3\mathbf{a} - 2\mathbf{b}$ to 1 decimal place.

Answers & Worked Solutions

1. $\begin{pmatrix} 6 \\ 4 \end{pmatrix} : \begin{pmatrix} 2+4 \\ 3+1 \end{pmatrix} = \begin{pmatrix} 6 \\ 4 \end{pmatrix}.$
2. $\begin{pmatrix} 4 \\ 2 \end{pmatrix} : \begin{pmatrix} 5+(-1) \\ -2+4 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}.$
3. $\begin{pmatrix} 4 \\ 6 \end{pmatrix} : \begin{pmatrix} 7-3 \\ 8-2 \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \end{pmatrix}.$
4. $\begin{pmatrix} -5 \\ 10 \end{pmatrix} : \begin{pmatrix} -3-2 \\ 6-(-4) \end{pmatrix} = \begin{pmatrix} -5 \\ 10 \end{pmatrix}.$
5. $\begin{pmatrix} 6 \\ -15 \end{pmatrix} : 3 \times \begin{pmatrix} 2 \\ -5 \end{pmatrix} = \begin{pmatrix} 3 \times 2 \\ 3 \times -5 \end{pmatrix}.$
6. $\begin{pmatrix} 8 \\ -6 \end{pmatrix} : -2 \times \begin{pmatrix} -4 \\ 3 \end{pmatrix} = \begin{pmatrix} -2 \times -4 \\ -2 \times 3 \end{pmatrix}.$
7. $\begin{pmatrix} 7 \\ 6 \end{pmatrix} : \begin{pmatrix} 6+1 \\ 8-2 \end{pmatrix} = \begin{pmatrix} 7 \\ 6 \end{pmatrix}.$
8. $\begin{pmatrix} -14 \\ 17 \end{pmatrix} : \begin{pmatrix} -6-8 \\ 15-(-2) \end{pmatrix} = \begin{pmatrix} -14 \\ 17 \end{pmatrix}.$
9. $5 : \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5.$
10. $13 : \sqrt{5^2 + (-12)^2} = \sqrt{25 + 144} = \sqrt{169} = 13.$
11. $\begin{pmatrix} 3 \\ 7 \end{pmatrix} : \begin{pmatrix} 5-2 \\ 4-(-3) \end{pmatrix} = \begin{pmatrix} 3 \\ 7 \end{pmatrix}.$
12. $\begin{pmatrix} 5 \\ 4 \end{pmatrix} : 2\mathbf{y} = \begin{pmatrix} 6+4 \\ 2+6 \end{pmatrix} = \begin{pmatrix} 10 \\ 8 \end{pmatrix}, \text{ so } \mathbf{y} = \begin{pmatrix} 5 \\ 4 \end{pmatrix}.$
13. $\begin{pmatrix} 3 \\ 4 \end{pmatrix} : \begin{pmatrix} 4-1 \\ 6-2 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}.$
14. $\begin{pmatrix} 5 \\ -12 \end{pmatrix} : \begin{pmatrix} 3-(-2) \\ -9-3 \end{pmatrix} = \begin{pmatrix} 5 \\ -12 \end{pmatrix}.$
15. $10 : \sqrt{(-6)^2 + 8^2} = \sqrt{36 + 64} = \sqrt{100} = 10.$
16. $\begin{pmatrix} 11 \\ -3 \end{pmatrix} : 2\begin{pmatrix} 1 \\ 2 \end{pmatrix} - \begin{pmatrix} -3 \\ 4 \end{pmatrix} + 3\begin{pmatrix} 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 2+3+6 \\ 4-4-3 \end{pmatrix} = \begin{pmatrix} 11 \\ -3 \end{pmatrix}.$
17. $5 : \text{Vector is } \begin{pmatrix} 3 \\ 4 \end{pmatrix}, \text{ magnitude is } \sqrt{3^2 + 4^2} = 5.$
18. $8 : \sqrt{k^2 + 6^2} = 10 \implies k^2 + 36 = 100 \implies k^2 = 64 \implies k = 8.$
19. $x = 3, y = -5 : x + 4 = 7 \implies x = 3; 3 + y = -2 \implies y = -5.$
20. $17.5 : 3\mathbf{a} - 2\mathbf{b} = \begin{pmatrix} 9-2 \\ -12-4 \end{pmatrix} = \begin{pmatrix} 7 \\ -16 \end{pmatrix}; \sqrt{7^2 + (-16)^2} = \sqrt{305} \approx 17.5.$