

Foundation GCSE: Congruence and Similarity

Name: _____

Class: _____

Instructions: Answer all questions. Questions are ordered from easiest (1) to hardest (20). Working out should be shown where appropriate.

1. State which condition (SSS, SAS, ASA, or RHS) proves that two right-angled triangles are congruent if they share a hypotenuse of length 10 cm and one side of length 6 cm.
2. Triangle A and Triangle B are mathematically similar. The scale factor of enlargement from Triangle A to Triangle B is 2. If a side on Triangle A has a length of 5 cm, calculate the length of the corresponding side on Triangle B.
3. Two mathematically similar rectangles have lengths of 4 cm and 12 cm respectively. Find the linear scale factor from the smaller rectangle to the larger rectangle.
4. A triangle has side lengths of 3 cm, 4 cm, and 5 cm. Another triangle has side lengths of 6 cm, 8 cm, and 10 cm. State the congruence condition or similarity status between them.
5. Two similar cylinders have heights of 3 cm and 9 cm respectively. Calculate the length scale factor.
6. Triangle P and Triangle Q are mathematically similar. The ratio of their corresponding side lengths is $\frac{2}{5}$. If a side on Triangle Q is 20 cm, find the length of the corresponding side on Triangle P.
7. Two similar shapes have a length scale factor of 3. If the area of the smaller shape is 12 cm^2 , calculate the area of the larger shape.
8. Two similar metal spheres have radii of 2 cm and 6 cm. Find the ratio of their surface areas in its simplest fractional form.
9. A photograph measuring 4 cm by 6 cm is enlarged mathematically similar to a width of 12 cm. Calculate the new height.
10. Two similar containers have volumes of 24 cm^3 and 192 cm^3 . Calculate the linear scale factor between the two containers.
11. Triangle ABC has angles 50° and 60° . Triangle DEF is mathematically similar to Triangle ABC and has an angle of 50° . State the remaining two interior angles of Triangle DEF.
12. Two similar solid blocks have a mass ratio of 1 : 8. Calculate the length scale factor between the two blocks.
13. A triangular garden patch has sides 5 m, 7 m, and 10 m. A scale drawing is made using a length scale factor of $\frac{1}{50}$. Find the length of the shortest side on the drawing in centimetres.
14. Two similar wooden cones have base radii of 4 cm and 10 cm. If the total surface area of the smaller cone is 80 cm^2 , calculate the surface area of the larger cone.
15. Two similar plastic models have heights of 5 cm and 15 cm. If the volume of the smaller model is 40 cm^3 , calculate the volume of the larger model.
16. Two similar jugs have surface areas of 50 cm^2 and 200 cm^2 . If the capacity of the smaller jug is 250 ml, calculate the capacity of the larger jug.
17. A map is drawn to a scale of 1 : 25000. If two landmarks are separated by 4.2 cm on the map, calculate the actual straight-line distance between them in kilometres.
18. Two similar iron bars have lengths of 12 cm and 36 cm. If the smaller bar has a weight of 160 g, calculate the weight of the larger bar.
19. Two similar pyramids have base areas of 18 cm^2 and 162 cm^2 . If the height of the smaller pyramid is 6 cm, calculate the height of the larger pyramid.
20. Two similar water tanks have a volume ratio of 27 : 125. If the total surface area of the smaller tank is 216 m^2 , calculate the surface area of the larger tank.

Answers & Worked Solutions

1. RHS : Right-angle, Hypotenuse, Side condition.
2. $10 \text{ cm} : 5 \times 2 = 10$.
3. $3 : \frac{12}{4} = 3$.
4. Similar : Side ratios are all equal to 2, proving mathematical similarity.
5. $3 : \frac{9}{3} = 3$.
6. $8 \text{ cm} : 20 \times \frac{2}{5} = 8$.
7. $108 \text{ cm}^2 : 12 \times 3^2 = 12 \times 9 = 108$.
8. $\frac{1}{9}$: Area scale factor is $\left(\frac{2}{6}\right)^2 = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$.
9. 18 cm : Scale factor is 3, so $6 \times 3 = 18$.
10. 2 : Volume ratio is $\frac{192}{24} = 8$, and $\sqrt[3]{8} = 2$.
11. 50° and 60° (or 50° and 70° , or 60° and 70°) : Corresponding angles in similar triangles are equal (third angle is 70°).
12. $2 : \sqrt[3]{8} = 2$.
13. $10 \text{ cm} : 5 \text{ m} = 500 \text{ cm}$, and $\frac{500}{50} = 10$.
14. 500 cm^2 : Area scale factor is $\left(\frac{10}{4}\right)^2 = 2.5^2 = 6.25$, so $80 \times 6.25 = 500$.
15. 1080 cm^3 : Length scale factor is 3, volume scale factor is $3^3 = 27$, so $40 \times 27 = 1080$.
16. 2000 ml : Area scale factor is $\frac{200}{50} = 4$, length scale factor is $\sqrt{4} = 2$, volume scale factor is $2^3 = 8$, so $250 \times 8 = 2000$.
17. $1.05 \text{ km} : 4.2 \times 25000 = 105000 \text{ cm} = 1050 \text{ m} = 1.05 \text{ km}$.
18. 4320 g : Length scale factor is 3, mass scale factor is $3^3 = 27$, so $160 \times 27 = 4320$.
19. 18 cm : Area scale factor is $\frac{162}{18} = 9$, length scale factor is $\sqrt{9} = 3$, so $6 \times 3 = 18$.
20. 600 m^2 : Volume ratio is $27 : 125$, length scale factor is $\sqrt[3]{\frac{125}{27}} = \frac{5}{3}$, area scale factor is $\left(\frac{5}{3}\right)^2 = \frac{25}{9}$, so $216 \times \frac{25}{9} = 600$.