

Foundation GCSE: Loci

Name: _____

Class: _____

Instructions: Answer all questions. Questions are ordered from easiest (1) to hardest (20). Working out should be shown where appropriate.

1. Describe the locus of points that are at a fixed distance of 3 cm from a single point P .
2. What construction gives the locus of points equidistant from two fixed points A and B ?
3. What construction gives the locus of points equidistant from two intersecting straight lines?
4. Describe the locus of points that are at a fixed distance of 2 cm from a straight line segment.
5. A dog is tethered to a post by a rope of length 4 m. What shape defines the boundary of the region the dog can reach?
6. Find the area of the locus of all points less than or equal to 5 cm away from a single point O . Give your answer in terms of π .
7. A treasure is buried closer to point A than point B . Which line divides the map into regions closer to A or B ?
8. Points must be within 3 m of a wall represented by a straight line of length 10 m. Ignoring the ends, calculate the area of this region on one side of the wall.
9. A goat is tied to a corner of a rectangular shed (6 m by 4 m) with a 3 m rope. Calculate the area the goat can graze. Give your answer to 1 decimal place.
10. A point moves so that it is always equidistant from points $X(2, 4)$ and $Y(8, 4)$. State the equation of the line representing this locus.
11. Find the perimeter of the region formed by points exactly 1 cm away from a line segment of length 6 cm (including semicircular ends). Give your answer to 2 decimal places.
12. On a map scale 1 cm : 10 m, a transmitter signal reaches up to 40 m. What is the radius in cm of the locus boundary drawn on the map?
13. A point moves so it is equidistant from the lines $y = x$ and $y = -x$. Name the geometric axes that represent this locus.
14. The locus of points at a distance r from a point P has a boundary length of 12π cm. Calculate r .
15. Points lie inside a square of side 8 cm and are less than 4 cm from a corner A . Calculate the area of this region to 1 decimal place.
16. Two light towers A and B are 50 m apart. A ship is always equidistant from both towers and 30 m from tower A . How far is the ship from tower B ?
17. Calculate the total area enclosed by the locus of points within 2 cm of a line segment of length 10 cm (including semicircular ends) in terms of π .
18. A point $P(x, y)$ is at a constant distance of 5 units from the origin $(0, 0)$. Write down the algebraic equation for this locus.
19. A lawn sprinkler sprays water in a circle of radius 6 m. If it is placed at the center of a square lawn of side 10 m, calculate the area of lawn that does NOT get watered to 1 decimal place.
20. Points A and B are 6 cm apart. Region R consists of points that are less than 4 cm from A and less than 4 cm from B . Describe the geometric boundary shape enclosing region R .

Answers & Worked Solutions

1. A circle of radius 3 cm centered at P .
2. Perpendicular bisector of the line segment AB .
3. Angle bisector of the angle between the two lines.
4. Two parallel lines 2 cm away on either side, connected by semicircles of radius 2 cm at the ends.
5. A circle of radius 4 m (or a disk/circular region).
6. $25\pi \text{ cm}^2$: Area = $\pi \times 5^2 = 25\pi$.
7. The perpendicular bisector of AB .
8. 30 m^2 : Rectangular region = $10 \times 3 = 30$.
9. 21.2 m^2 : Three-quarters of a circle of radius 3 m; Area = $\frac{3}{4} \times \pi \times 3^2 = 6.75\pi \approx 21.21 \approx 21.2$.
10. $x = 5$: Midpoint of X and Y along x -axis is $\frac{2+8}{2} = 5$. Perpendicular line is $x = 5$.
11. 18.28 cm : Two parallel lengths ($2 \times 6 = 12$) + two semicircular ends forming a full circle circumference of radius 1 cm ($2\pi \times 1 \approx 6.28$). Total = $12 + 6.28 = 18.28$.
12. 4 cm : $40 \div 10 = 4$.
13. The x -axis ($y = 0$) and y -axis ($x = 0$).
14. 6 cm : $2\pi r = 12\pi \implies r = 6$.
15. 12.6 cm^2 : Quarter circle of radius 4 cm; Area = $\frac{1}{4} \times \pi \times 4^2 = 4\pi \approx 12.57 \approx 12.6$.
16. 30 m : Equidistant from A and B means distance to B equals distance to A .
17. $40 + 4\pi \text{ cm}^2$: Rectangle = $10 \times (2 \times 2) = 40$; Two semicircles make a full circle of radius 2: area = $\pi \times 2^2 = 4\pi$. Total = $40 + 4\pi$.
18. $x^2 + y^2 = 25$: Circle equation $x^2 + y^2 = r^2$ with $r = 5$.
19. 4.9 m^2 : Total lawn area = $10^2 = 100 \text{ m}^2$; the circular spray of radius 6 m extends past the $10 \text{ m} \times 10 \text{ m}$ square sides, leaving four unwatered corners. The watered area inside the square is approximately 95.1 m^2 , leaving $100 - 95.1 = 4.9 \text{ m}^2$ unwatered.
20. A vesica piscis (or symmetrical lens shape formed by two intersecting circular arcs).