Foundation GCSE: Prime Numbers

Name: _____ Class: _____

Instructions: Answer all questions. Questions are ordered from easiest (1) to hardest (20). Working out should be shown where appropriate.

1. Write down the smallest prime number.
2. Circle the prime numbers in this list: $1, 2, 5, 9, 13, 15$.
3. Write down all the prime numbers between 1 and 10.
4. Explain why 9 is not a prime number.
5. Write down the only even prime number.
6. Find all the prime numbers between 20 and 30.
7. Express 20 as the sum of two prime numbers.
8. True or false: All prime numbers greater than 3 are odd.
9. Write 48 as a product of its prime factors in index form.
10. Find the prime number that is closest to 50.
11. Express 180 as a product of prime factors.
12. Find two prime numbers whose sum is 30 and whose difference is 4.
13. Write 300 as a product of its prime factors in index form.
14. Given that $54 = 2 \times 3^3$, find the smallest integer k such that $54k$ is a square number.
15. Find the smallest prime number greater than 100.
16. Express 504 as a product of prime factors in index form.
17. Find the value of p if the prime factorisation of $1\,260$ is $2^2 \times 3^2 \times p \times 7$.
18. Determine whether 119 is a prime number. Show working.
19. Find three prime numbers whose sum is 41.
20. Find the smallest integer n such that $2\,205 \times n$ is a cube number.

Answers & Worked Solutions

1. 2

   2 *is the smallest and only even prime number.*

2. $2, 5, 13$

   1 *is not prime;* 9 *and* 15 *have factors other than* 1 *and themselves.*

3. $2, 3, 5, 7$

   Numbers with exactly two distinct factors.

4. It has more than two factors $(1, 3, 9)$.

   Prime numbers must have exactly two factors.

5. 2

   All other even numbers are divisible by 2*.*

6. $23, 29$

   Checking integers between 20 *and* 30*.*

7. $3 + 17$ (or $7 + 13$)

   Valid pairs of primes adding to 20*.*

8. True

   Any even number greater than 2 *would be divisible by* 2*.*

9. $2^4 \times 3$

   $48 \div 2 = 24$*, continuing prime division gives* $2^4 \times 3$*.*

10. 47 (or 53)

    Both 47 *and* 53 *are prime;* 47 *is* 3 *away,* 53 *is* 3 *away (either acceptable).*

11. $2^2 \times 3^2 \times 5$

    $180 = 18 \times 10 = (2 \times 3^2) \times (2 \times 5)$.

12. 13 and 17

    $13 + 17 = 30$ *and* $17 - 13 = 4$*.*

13. $2^2 \times 3 \times 5^2$

    $300 = 3 \times 100 = 3 \times 2^2 \times 5^2$*.*

14. 6

    $54k = 2 \times 3^3 \times k$*; need even powers, so* $k = 2 \times 3 = 6$*.*

15. 101

    Check prime divisors up to $\sqrt{101} \approx 10$*: not divisible by* 2*,* 3*, or* 5*. Checking* 7 *gives* $101 \div 7 = 14$ *remainder* 3*. Thus,* 101 *is prime.*

16. $2^3 \times 3^2 \times 7$

    $504 \div 2 = 252$*, breaking down gives* $2^3 \times 3^2 \times 7$*.*

17. 5

    $1\,260 \div (4 \times 9 \times 7) = 1\,260 \div 252 = 5$.

18. No $(119 = 7 \times 17)$

    Testing primes up to $\sqrt{119} \approx 10.9$*, find* $7 \times 17 = 119$*.*

19. $3, 7$, and 31

    *Since their sum is odd (*41*), all three primes must be odd (trying* 3 *leaves* 38*, satisfied by* 7 *and* 31*).*

20. 525

    $2\,205 = 3^2 \times 5^1 \times 7^2$*. To make all powers multiples of* 3*, we need* $3^{3-2} \times 5^{3-1} \times 7^{3-2} = 3 \times 25 \times 7 = 525$*.*