Foundation GCSE: Vectors Problems

Name: ______________________ Class: __________

Instructions: Answer all questions. Questions are ordered from easiest (1) to hardest (20). Working out should be shown where appropriate.

1. In triangle OAB, $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$. Write down the vector $\overrightarrow{AB}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

2. M is the midpoint of line segment AB. If $\overrightarrow{AB} = \begin{pmatrix} 6 \\ -8 \end{pmatrix}$, find the column vector $\overrightarrow{AM}$.

3. Vector $\mathbf{a} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ and vector $\mathbf{b} = \begin{pmatrix} 6 \\ k \end{pmatrix}$. Given that $\mathbf{a}$ and $\mathbf{b}$ are parallel, find the value of k.

4. In parallelogram $OPQR$, $\overrightarrow{OP} = \mathbf{p}$ and $\overrightarrow{OR} = \mathbf{r}$. Express vector $\overrightarrow{PR}$ in terms of $\mathbf{p}$ and $\mathbf{r}$.

5. Points A, B, and C are on a straight line. $\overrightarrow{AB} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ and $\overrightarrow{BC} = \begin{pmatrix} 4 \\ 6 \end{pmatrix}$. Show that AB and BC are parallel and state the ratio $AB : BC$.

6. In triangle XYZ, $\overrightarrow{XY} = 2\mathbf{a}$ and $\overrightarrow{YZ} = 3\mathbf{b}$. Point M is the midpoint of XZ. Find $\overrightarrow{YM}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

7. Given that vector $\mathbf{x} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$ and $\mathbf{y} = \begin{pmatrix} -4 \\ 2 \end{pmatrix}$, explain whether $\mathbf{x}$ and $\mathbf{y}$ are parallel.

8. A quadrilateral $OABC$ is a parallelogram where $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$. M is the midpoint of AC. Express $\overrightarrow{OM}$ in terms of $\mathbf{a}$ and $\mathbf{c}$.

9. Vector $\mathbf{p} = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$. Find the scalar k such that $k\mathbf{p} = \begin{pmatrix} 12 \\ -9 \end{pmatrix}$.

10. In triangle ABC, $\overrightarrow{AB} = 3\mathbf{u}$ and $\overrightarrow{BC} = 6\mathbf{v}$. Point D lies on AC such that $AD : DC = 1 : 2$. Express $\overrightarrow{BD}$ in terms of $\mathbf{u}$ and $\mathbf{v}$.

11. Vector $\mathbf{a} = \begin{pmatrix} m-1 \\ 5 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 3 \\ 3 \end{pmatrix}$. If $\mathbf{a}$ is parallel to $\mathbf{b}$, find the value of m.

12. In triangle OPQ, M is the midpoint of OP and N is the midpoint of OQ. If $\overrightarrow{OP} = 2\mathbf{p}$ and $\overrightarrow{OQ} = 2\mathbf{q}$, express $\overrightarrow{MN}$ in terms of $\mathbf{p}$ and $\mathbf{q}$.

13. $ABCD$ is a trapezium where $\overrightarrow{AB} = 2\mathbf{a}$, $\overrightarrow{DC} = 6\mathbf{a}$, and $\overrightarrow{AD} = \mathbf{b}$. Find $\overrightarrow{BC}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

14. Points P, Q, and R have position vectors $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$, $\begin{pmatrix} 4 \\ 5 \end{pmatrix}$, and $\begin{pmatrix} 7 \\ 8 \end{pmatrix}$ respectively. Prove that P, Q, and R are collinear.

15. In triangle OAB, $\overrightarrow{OA} = 6\mathbf{a}$ and $\overrightarrow{OB} = 9\mathbf{b}$. Point P lies on AB such that $AP : PB = 1 : 2$. Express $\overrightarrow{OP}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

16. In parallelogram $OABC$, $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$. Point X divides OB in the ratio $1 : 3$. Express $\overrightarrow{OX}$ in terms of $\mathbf{a}$ and $\mathbf{c}$.

17. Vector $\mathbf{v} = \begin{pmatrix} 5 \\ -12 \end{pmatrix}$. Find the vector of length 26 in the opposite direction to $\mathbf{v}$.

18. In triangle ABC, $\overrightarrow{AB} = \mathbf{a}$ and $\overrightarrow{AC} = \mathbf{b}$. Point M is on BC such that $BM : MC = 3 : 1$. Express $\overrightarrow{AM}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

19. Triangle OPQ has $\overrightarrow{OP} = \mathbf{p}$ and $\overrightarrow{OQ} = \mathbf{q}$. Point X is on PQ such that $PX : XQ = 2 : 1$. Point Y is on OQ such that $OY : YQ = 1 : 2$. Express $\overrightarrow{XY}$ in terms of $\mathbf{p}$ and $\mathbf{q}$.

20. In triangle OAB, $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$. Point M divides OA in the ratio $2 : 1$ and point N divides OB in the ratio $1 : 3$. Express $\overrightarrow{MN}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

Answers & Worked Solutions

1. $\mathbf{b}-\mathbf{a}$: $\overrightarrow{AB}=\overrightarrow{AO}+\overrightarrow{OB}=-\mathbf{a}+\mathbf{b}$.
2. $\begin{pmatrix}3\\-4\end{pmatrix}$: $\frac{1}{2}\overrightarrow{AB}=\begin{pmatrix}6\div 2\\-8\div 2\end{pmatrix}=\begin{pmatrix}3\\-4\end{pmatrix}$.
3. $k=8$: $\mathbf{b}=2\mathbf{a}$, so $k=2\times 4=8$.
4. $\mathbf{r}-\mathbf{p}$: $\overrightarrow{PR}=\overrightarrow{PO}+\overrightarrow{OR}=-\mathbf{p}+\mathbf{r}$.
5. Parallel, ratio 1 : 2 : $\overrightarrow{BC}=2\overrightarrow{AB}$, so they are parallel and the ratio is 1 : 2.
6. $-\mathbf{a}+\frac{3}{2}\mathbf{b}$: $\overrightarrow{XZ}=2\mathbf{a}+3\mathbf{b}$; $\overrightarrow{YM}=-2\mathbf{a}+\frac{1}{2}(2\mathbf{a}+3\mathbf{b})=-\mathbf{a}+\frac{3}{2}\mathbf{b}$.
7. Yes, parallel : $\mathbf{y}=-2\mathbf{x}$, so the vectors are scalar multiples.
8. $\frac{1}{2}(\mathbf{a}+\mathbf{c})$: $\overrightarrow{OM}=\mathbf{a}+\frac{1}{2}\overrightarrow{AC}=\mathbf{a}+\frac{1}{2}(\mathbf{c}-\mathbf{a})=\frac{1}{2}(\mathbf{a}+\mathbf{c})$.
9. $k=3$: $\begin{pmatrix}12\\-9\end{pmatrix}=3\begin{pmatrix}4\\-3\end{pmatrix}$.
10. $2\mathbf{v}-4\mathbf{u}$: $\overrightarrow{AC}=6\mathbf{v}-3\mathbf{u}$; $\overrightarrow{AD}=\frac{1}{3}\overrightarrow{AC}=2\mathbf{v}-\mathbf{u}$; $\overrightarrow{BD}=-3\mathbf{u}+(2\mathbf{v}-\mathbf{u})=2\mathbf{v}-4\mathbf{u}$.
11. $m=6$: $\frac{m-1}{3}=\frac{5}{3}\implies m-1=5\implies m=6$.
12. $\mathbf{q}-\mathbf{p}$: $\overrightarrow{MN}=\overrightarrow{ON}-\overrightarrow{OM}=\mathbf{q}-\mathbf{p}$.
13. $4\mathbf{a}+\mathbf{b}$: $\overrightarrow{BC}=\overrightarrow{BA}+\overrightarrow{AD}+\overrightarrow{DC}=-2\mathbf{a}+\mathbf{b}+6\mathbf{a}=4\mathbf{a}+\mathbf{b}$.
14. Collinear : $\overrightarrow{PQ}=\begin{pmatrix}3\\3\end{pmatrix}$ and $\overrightarrow{QR}=\begin{pmatrix}3\\3\end{pmatrix}$. Equal vectors sharing Q implies a straight line.
15. $4\mathbf{a}+3\mathbf{b}$: $\overrightarrow{AB}=9\mathbf{b}-6\mathbf{a}$; $\overrightarrow{AP}=\frac{1}{3}(9\mathbf{b}-6\mathbf{a})=3\mathbf{b}-2\mathbf{a}$; $\overrightarrow{OP}=6\mathbf{a}+(3\mathbf{b}-2\mathbf{a})=4\mathbf{a}+3\mathbf{b}$.
16. $\frac{1}{4}(\mathbf{a}+\mathbf{c})$: $\overrightarrow{OB}=\mathbf{a}+\mathbf{c}$, so $\overrightarrow{OX}=\frac{1}{4}(\mathbf{a}+\mathbf{c})$.
17. $\begin{pmatrix}-10\\24\end{pmatrix}$: Magnitude of $\mathbf{v}$ is 13; opposite vector of length 26 is $-2\mathbf{v}=\begin{pmatrix}-10\\24\end{pmatrix}$.
18. $\frac{1}{4}\mathbf{a}+\frac{3}{4}\mathbf{b}$: $\overrightarrow{BC}=\mathbf{b}-\mathbf{a}$; $\overrightarrow{AM}=\mathbf{a}+\frac{3}{4}(\mathbf{b}-\mathbf{a})=\frac{1}{4}\mathbf{a}+\frac{3}{4}\mathbf{b}$.
19. $-\frac{1}{3}\mathbf{p}-\frac{1}{3}\mathbf{q}$: $\overrightarrow{OX}=\frac{1}{3}\mathbf{p}+\frac{2}{3}\mathbf{q}$; $\overrightarrow{OY}=\frac{1}{3}\mathbf{q}$; $\overrightarrow{XY}=\overrightarrow{OY}-\overrightarrow{OX}=-\frac{1}{3}\mathbf{p}-\frac{1}{3}\mathbf{q}$.
20. $\frac{1}{4}\mathbf{b}-\frac{2}{3}\mathbf{a}$: $\overrightarrow{OM}=\frac{2}{3}\mathbf{a}$ and $\overrightarrow{ON}=\frac{1}{4}\mathbf{b}$, so $\overrightarrow{MN}=\overrightarrow{ON}-\overrightarrow{OM}=\frac{1}{4}\mathbf{b}-\frac{2}{3}\mathbf{a}$.